Lecture 2
Symmetric Encryption I

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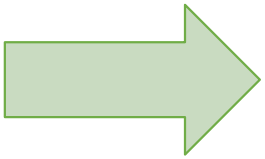
Plan



1. Computational definitions of security
2. If semantically-secure encryption exists then **P ≠ NP**
3. A proof that "the PRGs imply secure encryption"
4. Theoretical constructions of PRGs
5. Stream ciphers

How to change the security definition?

we will require that m_0, m_1 are chosen by a **poly-time adversary**

An encryption scheme is **perfectly secret** if for every $m_0, m_1 \in \mathcal{M}$

$\mathrm{Enc}(K, m_0)$ and $\mathrm{Enc}(K, m_1)$ are identically distributed

we will require that no **poly-time adversary** can distinguish $\mathrm{Enc}(K, m_0)$ from $\mathrm{Enc}(K, m_1)$

A game

(Enc,Dec) – an encryption scheme

security parameter
1^n

adversary
(polynomial-time probabilistic Turing machine)

oracle

chooses m_0, m_1 such that
$|m_0| = |m_1|$

m_0, m_1

1. selects k randomly from $\{0,1\}^n$
2. chooses a random $b = 0,1$
3. calculates
   $c := \mathrm{Enc}(k, m_b)$

has to guess b

c

Alternative name: **has indistinguishable encryptions**

Security definition:
We say that **(Enc,Dec)** is **semantically-secure** if any **polynomial time** adversary guesses b correctly with probability at most $\frac{1}{2} + \varepsilon(n)$, where ε is negligible.

Testing the definition

Suppose the adversary can compute ***k*** from **Enc(*k*,*m*)**. Can he win the game?

YES!

Suppose the adversary can compute **some bit of *m*** from **Enc(*k*,*m*)**. Can he win the game?

YES!

Multiple messages

In real-life applications we need to encrypt **multiple messages with one key**.

The adversary may learn something about the key by looking at

ciphertexts $c_1,\ldots,c_t$ of

some messages $m_1,\ldots,m_t$.

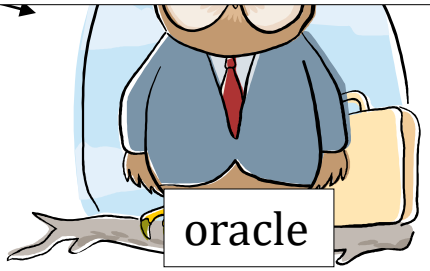
How are these messages chosen?
let's say: the adversary can choose them!

(good tradition: be as pessimistic as possible)

A chosen-plaintext attack (CPA)

security parameter 1^n

1. chooses random $k \leftarrow \{0,1\}^n$
2. chooses random $b \leftarrow \{0,1\}$



chooses m'_1

m'_1

$c_1 = \mathrm{Enc}(k, m'_1)$

$\vdots$

chooses m'_t

m'_t

$c_t = \mathrm{Enc}(m'_t)$

challenge phase:

chooses m_0, m_1

m_0, m_1

$c = \mathrm{Enc}(k, m_b)$

the interaction continues . . .

has to guess b

CPA-security

Alternative name: CPA-secure

Security definition

We say that **(Enc,Dec) has indistinguishable encryptions under a chosen-plaintext attack (CPA)** if

every **randomized polynomial time** adversary
guesses $\boldsymbol{b}$ correctly
with probability at most $\frac{1}{2} + \boldsymbol{\varepsilon}(\boldsymbol{n})$, where $\boldsymbol{\varepsilon}$ is negligible.

Observation

A **CPA-secure** encryption scheme cannot be deterministic.

Typical options:

- **Enc** has a "state" (e.g. a counter)
- **Enc** is randomized, i.e., it takes as additional input:
  - some perfect randomness ***R***, or
  - takes as an **nonce *R***

weaker requirement

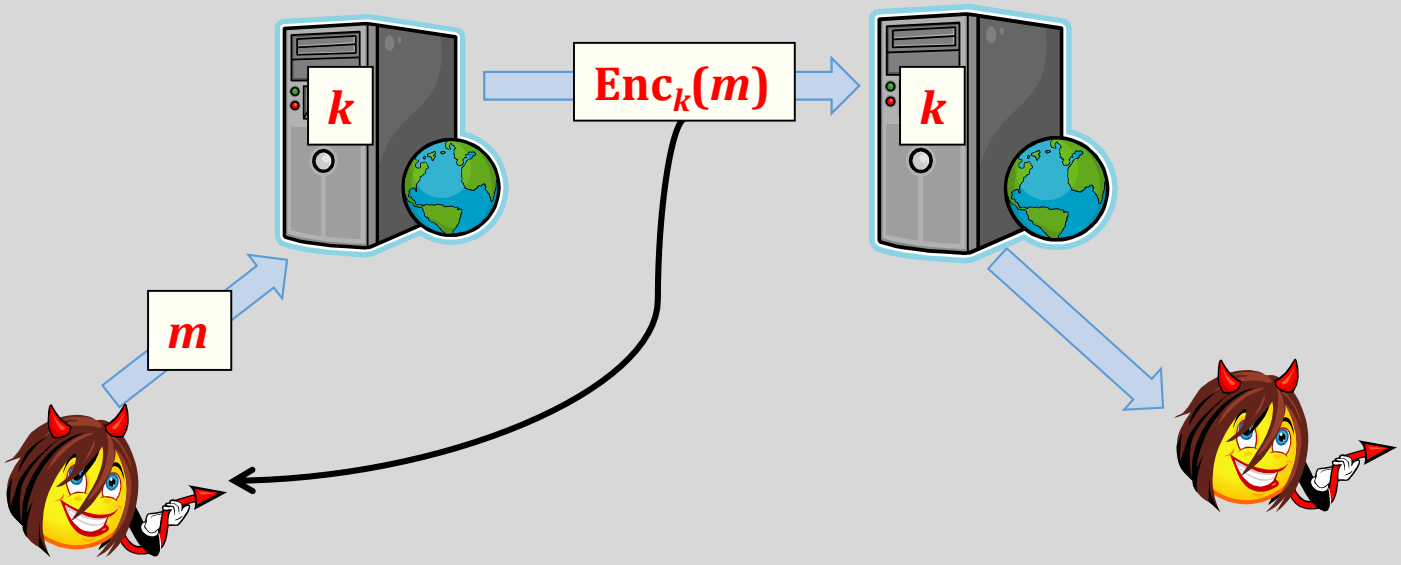
nonce = "**n**umber used **once**"

CPA in real-life

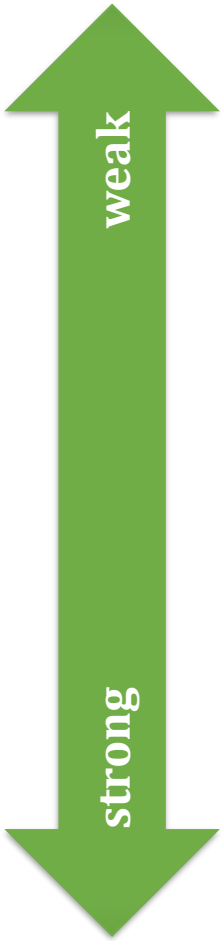
Q: Aren't we too pessimistic?

A: **No!** **CPA** can be implemented in practice.

Example: routing



Other attacks known in the literature

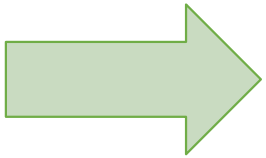


- **ciphertext-only attack** – the adversary has no information about the plaintext
- **known plaintext attack** – the plaintext are drawn from some distribution that the adversary does not control
- **batch chosen-plaintext attack** – like the **CPA** attack, but the adversary has to choose $m_1,\ldots,m_t$ at once.

("our" **CPA**-attack is also called the "**adaptive CPA-attack**")

- **chosen ciphertext attack** – we will discuss it later…

Plan



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Is it possible to prove security?

Bad news:

Theorem

If semantically-secure encryption exists (with $|k| < |m|$)

P ≠ NP

Intuition: if **P = NP** then the adversary can guess the key...

(formal proof – exercises)

Moral:
"If **P=NP**, then the semantically-secure encryption is broken"

Is it 100% true?

Not really...

This is because even if **P=NP** we do not know what are the constants.

Maybe **P=NP** in a very "inefficient way"...

To prove security of a cryptographic scheme we need to show **a lower bound on the computational complexity of some problem.**

In the "asymptotic setting" that would mean that at least we show that **P ≠ NP**.

Does the implication in the other direction hold? (that is: does **P ≠ NP** imply anything for cryptography?)

No! (at least as far as we know)

Therefore

proving that an encryption scheme is secure is probably much harder than proving that **P ≠ NP**.

What can we prove?

We can prove conditional results.

That is, we can show theorems of a type:

Suppose that some "computational assumption **A**" holds

⇓

then scheme **X** is secure.

Suppose that some scheme **Y** is secure

⇓

then scheme **X** is secure.

A natural question

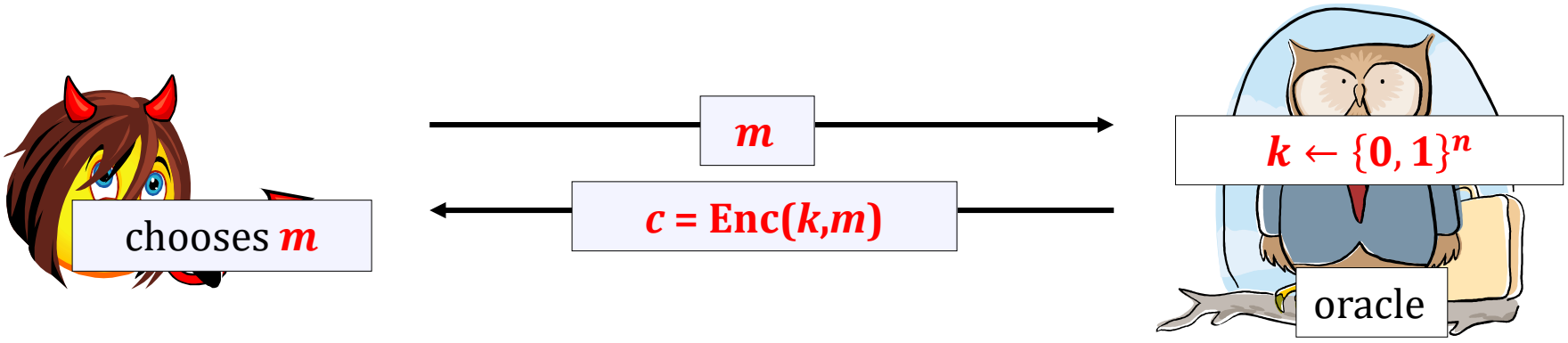
What is "**the minimal assumption**" needed for cryptography?

Answer: existence **one-way functions**.

We introduce them now.

Why **P ≠ NP** is not enough?

Intuition:



It may be that:

- it is **NP-hard** to compute k from $c = \mathrm{Enc}(k,m)$,
- but it is **easy** on average.

The "minimal assumption for cryptography"?

In the scenario from the previous slide we need that:

for a fixed m the key k is hard to compute from

$$c = \mathbf{Enc}(k,m)$$

if k is random

look at it as: a **fixed** function f of k
defined as $f(k) = \mathbf{Enc}(k, m)$

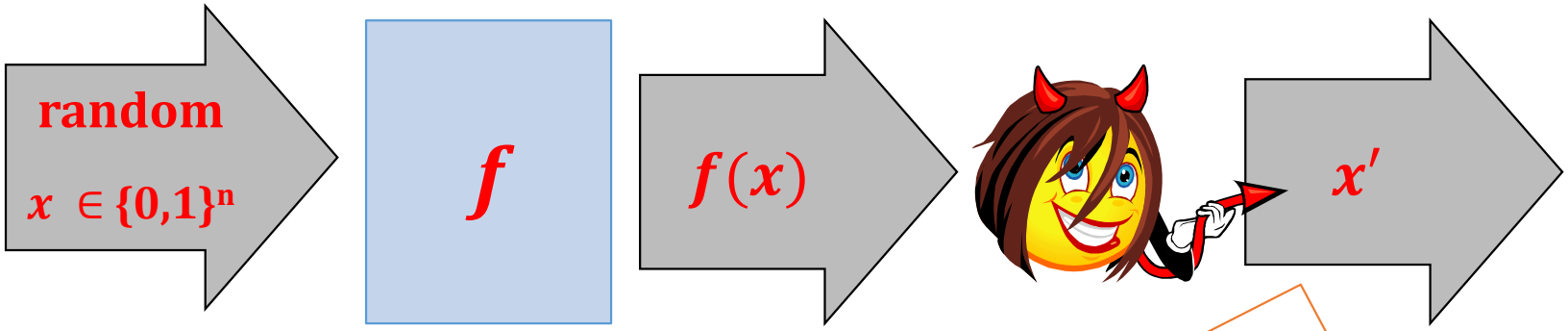
we need that
k is hard to compute from $f(k)$.
Such f is called **one-way**.

One-way functions

A function

$$f: \{0,1\}^* \to \{0,1\}^*$$

is **one-way** if it is: **(1)** poly-time computable, and **(2)** "hard to invert it".



probability that any poly-time adversary outputs x' such that

$$f(x) = f(x')$$

is negligible in n.

A real-life analogue: phone book

A function:

people → numbers

is “one way”.

More formally...

experiment **(machine M, function f)**

1. pick a random element $x \leftarrow \{0,1\}^n$
2. let $y := f(x)$,
3. let x' be the output of M on y
4. we say that M **won** if $f(x') = y$.

We will say that a poly-time computable $f: \{0,1\}^* \to \{0,1\}^*$ is **one-way** if

$\forall$ **polynomial-time Turing Machine M**

P (M wins) is negligible

Example of a (candidate for) a one-way function

If **P=NP** then **one-way functions don't exist**.

Therefore currently no function can be proven to be one-way.

But there exist candidates.

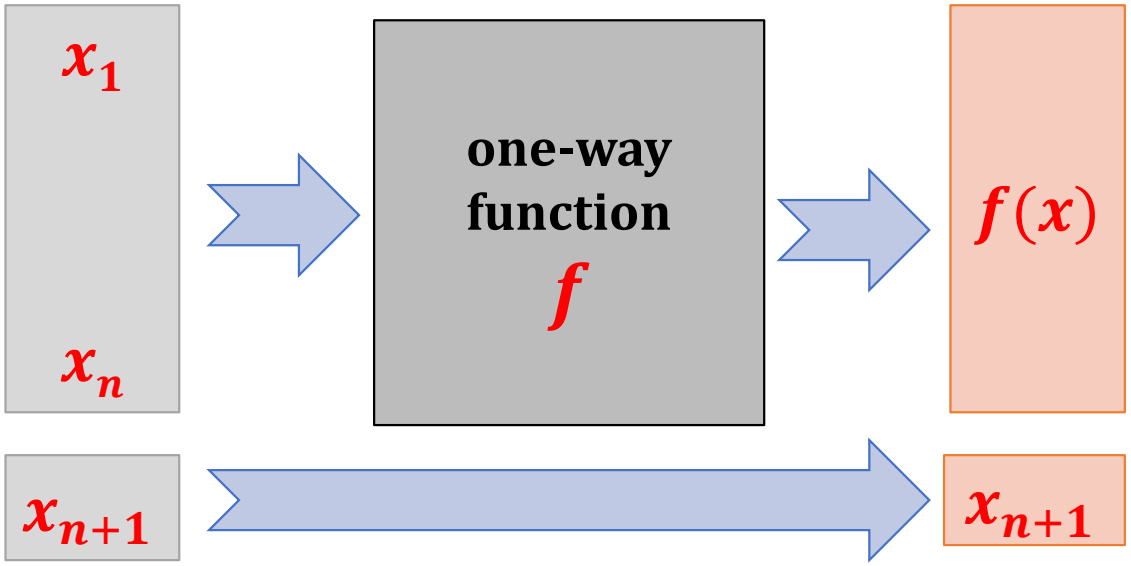
Example:

$f(p,q) = pq$, where p and q are primes such that $|p| = |q|$.

this function is defined on
primes × primes,
not on
$\{0,1\}^*$
but it's just a technicality

One way functions **do not** "hide all the input"

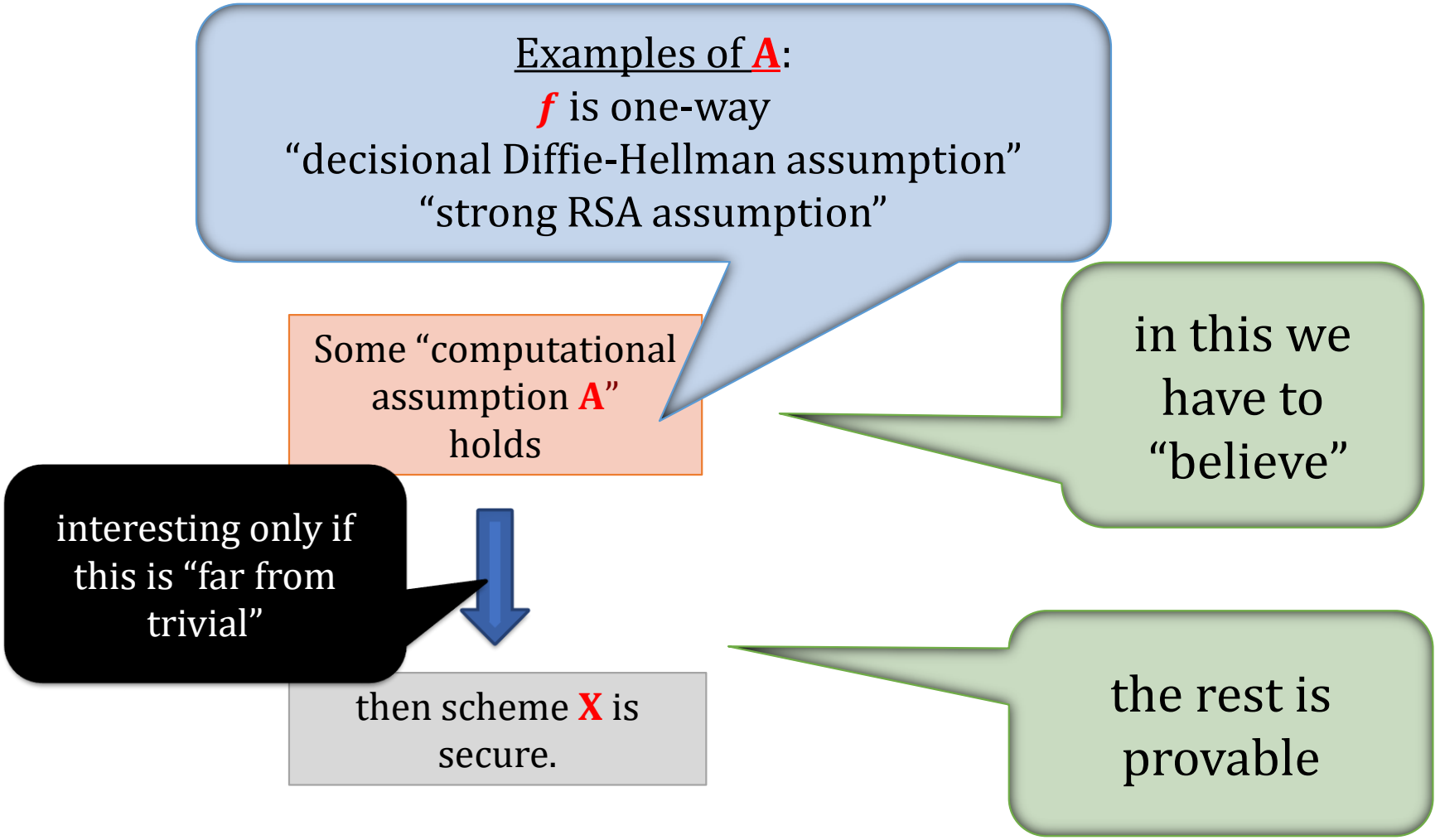
Example:



$f'(x_1, \ldots, x_{n+1}) := f(x_1, \ldots, x_n) \,||\, x_{n+1}$ is also a one-way function

Research program in cryptography

Base the security of cryptographic schemes on a small number of well-specified "computational assumptions".

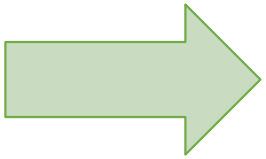


Example

Suppose that G is a "cryptographic pseudorandom generator"

we can construct a secure encryption scheme based on G

Plan



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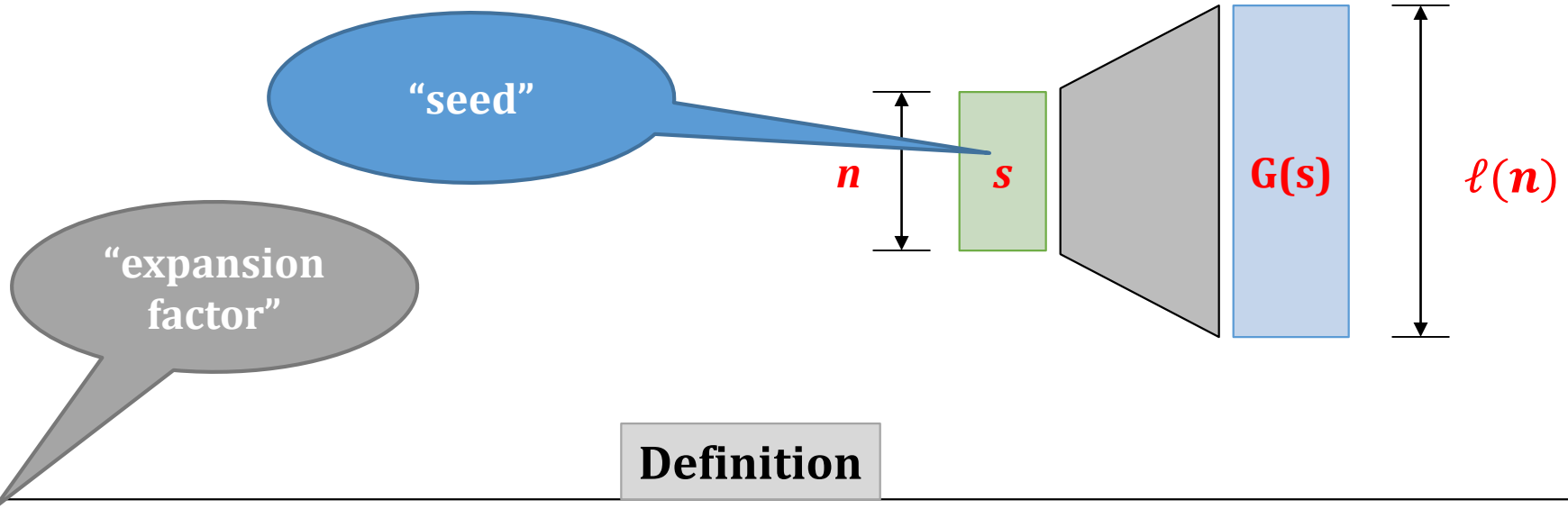
Pseudorandom generators



Definition

ℓ – polynomial such that always $\ell(n) > n$

An algorithm $G : \{0,1\}^* \to \{0,1\}^*$ is called a **pseudorandom generator (PRG)** if

for every n

and for every s such that $|s| = n$

we have

$$|G(s)| = \ell(n).$$

this has to be formalized

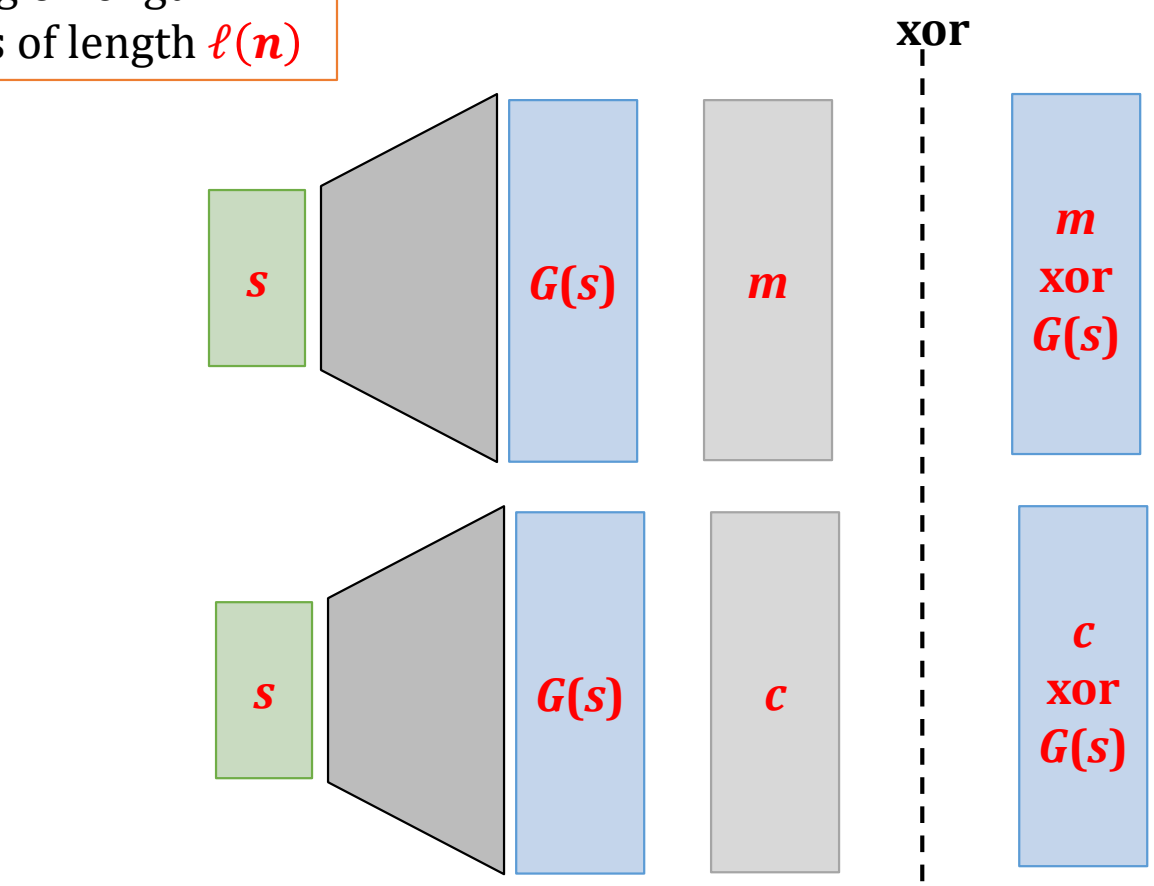
and for a random s the value $G(s)$ "**looks random**".

Idea

Use PRGs to "shorten" the key in the one time pad

for a moment just consider a **single message case**

Key: random string of length n
Plaintexts: strings of length $\ell(n)$



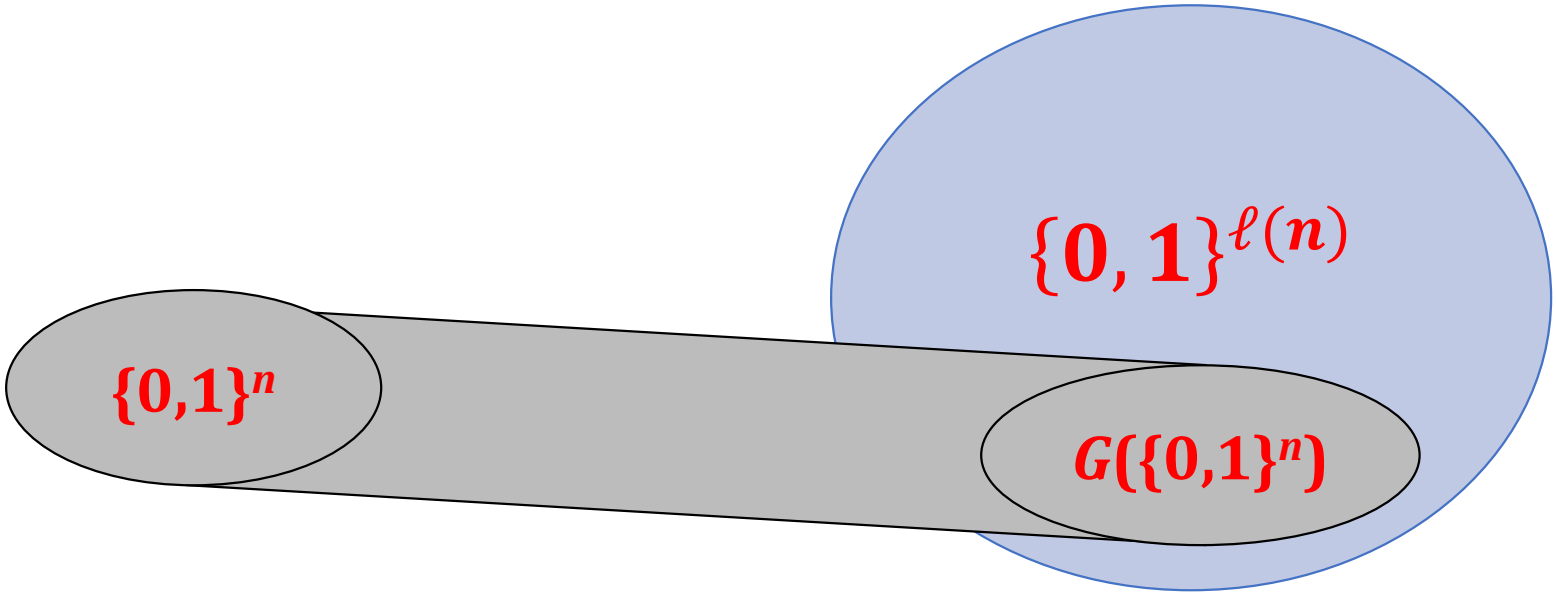
If we use a "normal PRG" – this idea doesn't work
We have to use the **cryptographic PRGs.**

“Looks random”

Suppose $s \in \{0,1\}^n$ is chosen randomly.

Can $G(s) \in \{0,1\}^{\ell(n)}$ be uniformly random?

No!



“Looks random”

What does it mean?

Non-cryptographic applications:

should pass **some statistical tests**.

Cryptography:

should pass **all polynomial-time tests**.

Non-cryptographic PRGs

Example: **Linear Congruential Generators (LCG)** defined recursively

- $X_0 \in Z_m$ – the key
- for $n = 1, 2, \ldots$ let

$$X_{n+1} := (a \cdot X_n + c) \bmod m$$

output: $Y_1, Y_2, \ldots$ where

Y_i = first t bits of each X_i

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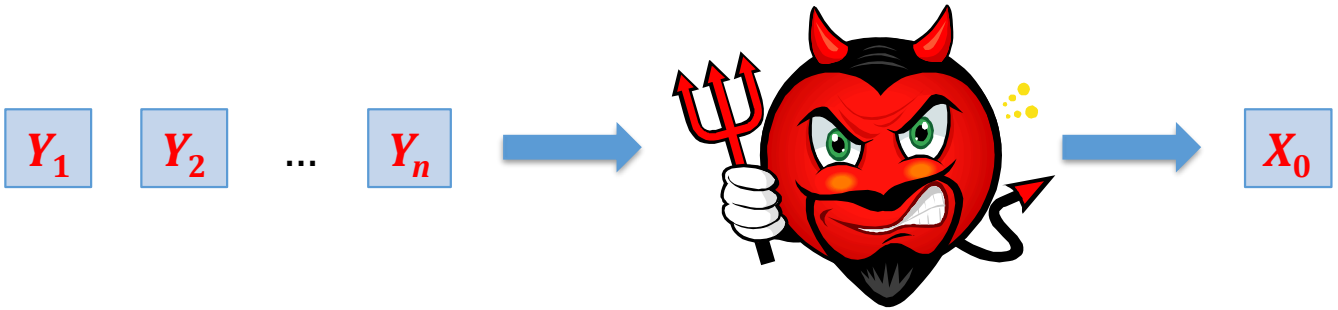
rand() function in Windows – an **LCG** with

$$a = 214013,\ c = 2531011,\ m = 2^{32},\ t = 15$$

How to break an LRS



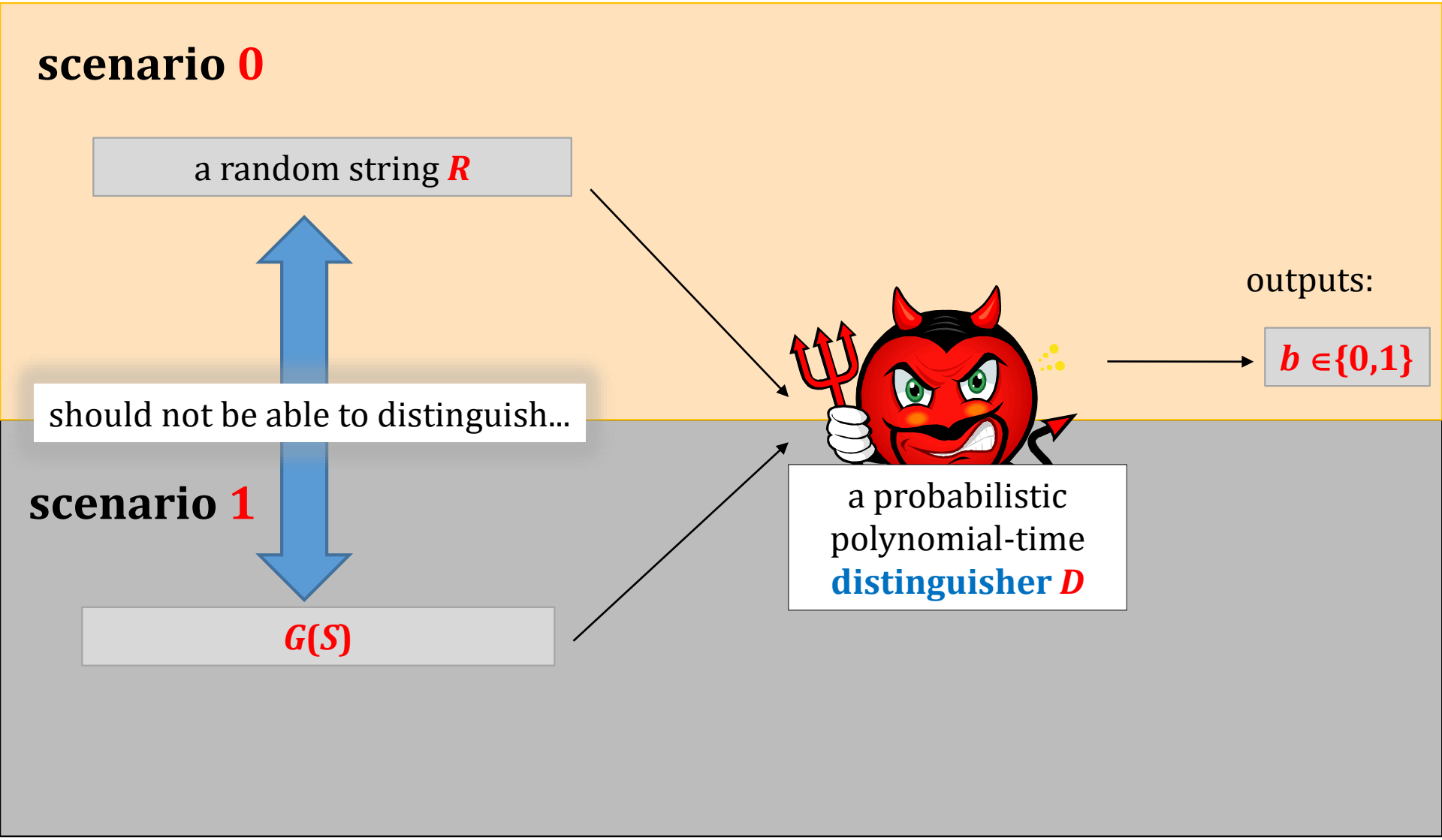
Solve linear equations with "partial knowledge" (because you only know only first ***t*** bits)

See:

G. Argyros and A. Kiayias: **I Forgot Your Password: Randomness Attacks Against PHP Applications**, *USENIX Security '12*

(successful attacks on password-recovery mechanisms in PHP)

PRG – main idea of the definition



Cryptographic PRG

a random string R

or

$G(S)$ (where S random)

outputs:

0 if he thinks it's R

1 if he thinks it's $G(S)$

Should not be able to distinguish...

Definition

n – a parameter

S – a variable distributed uniformly over $\{0,1\}^n$

R – a variable distributed uniformly over $\{0,1\}^{\ell(n)}$

G is a **cryptographic PRG** if

for every polynomial-time Turing Machine D

we have that

$$\left|P(D(R)=1)-P\left(D\left(G(S)\right)=1\right)\right|$$

is negligible in n.

Constructions

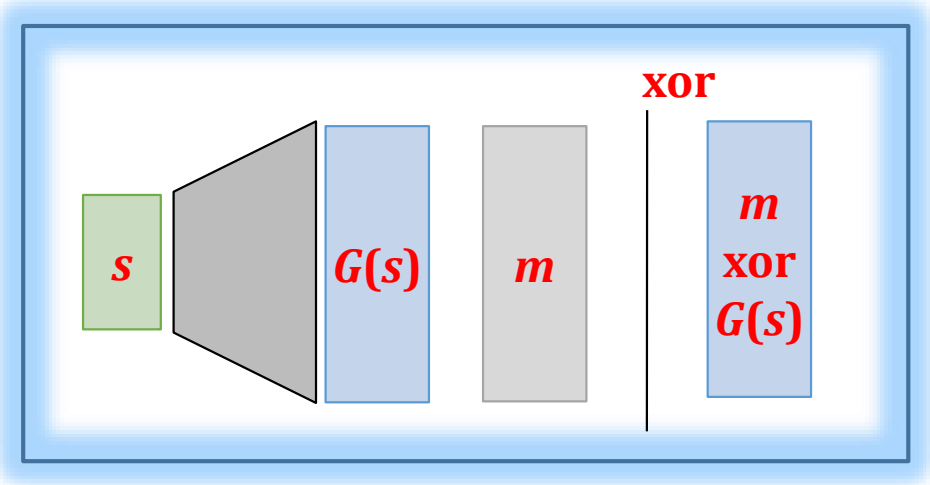
There exists constructions of cryptographic pseudorandom-generators, that are **conjectured** to be secure.

We will discuss them later...

Theorem

(for simplicity consider only the single message case)

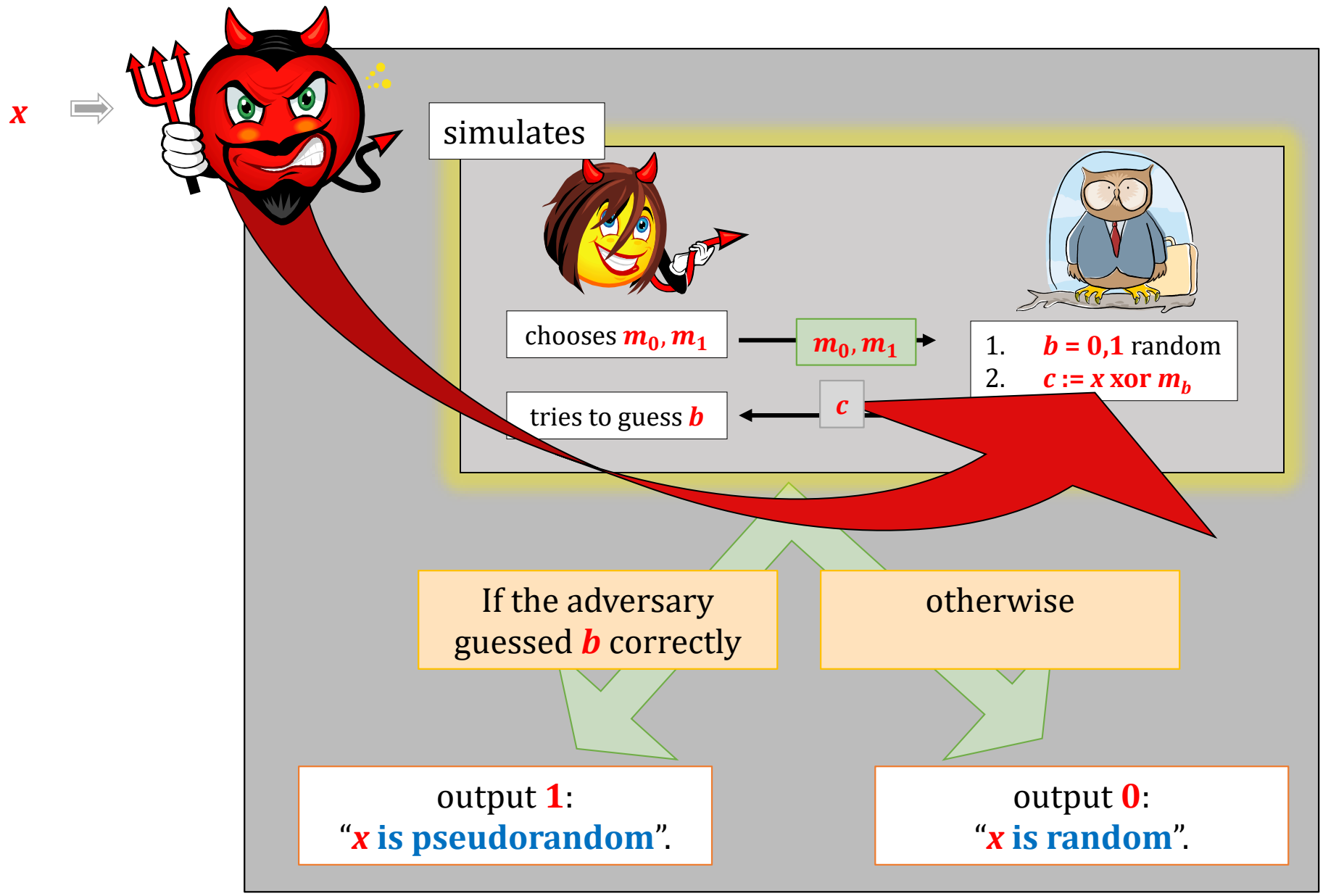
If **G** is a **cryptographic PRG** then the encryption scheme constructed before is semantically secure.



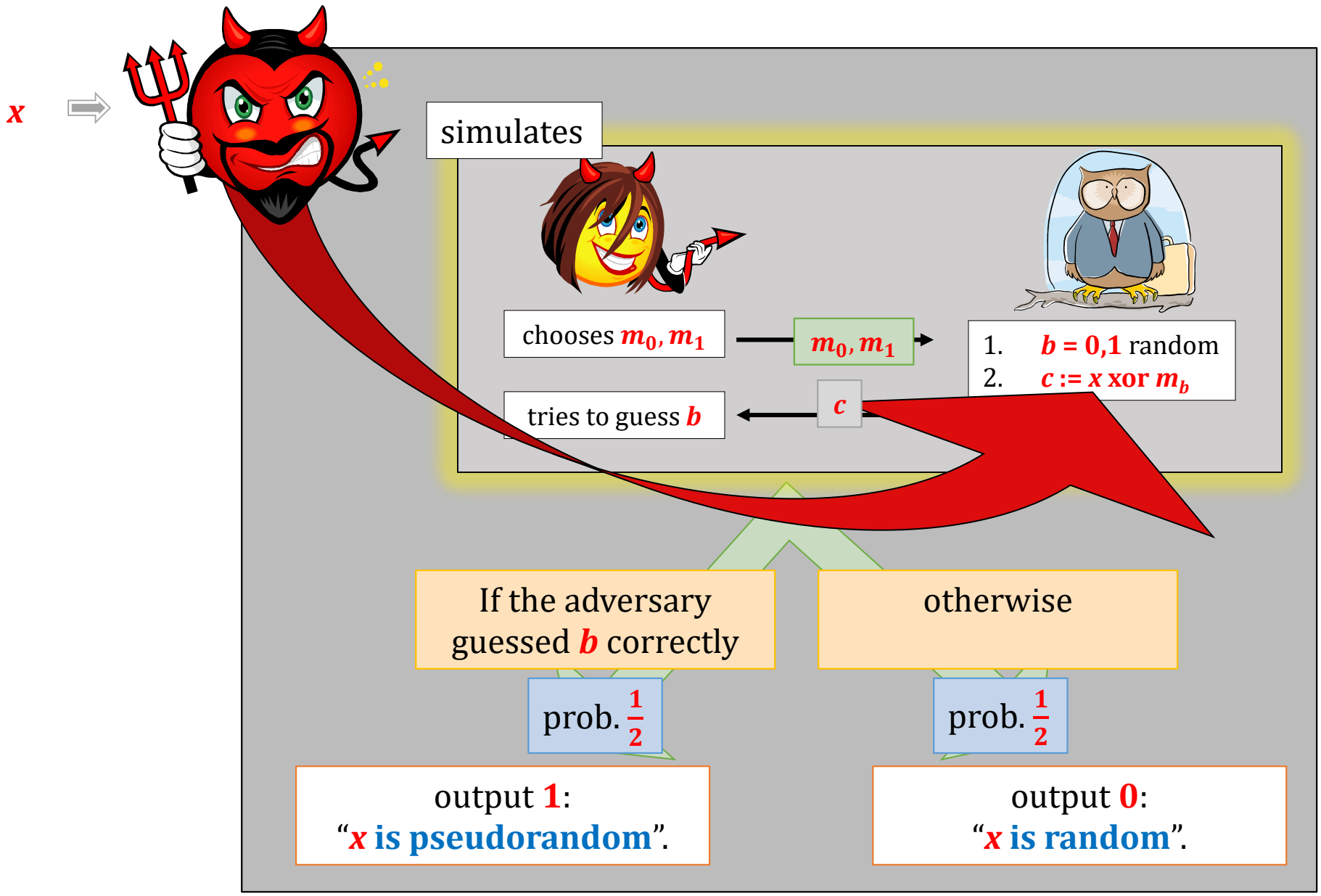
cryptographic PRGs exist ⇒ CPA-secure encryption exists

Proof (sketch)

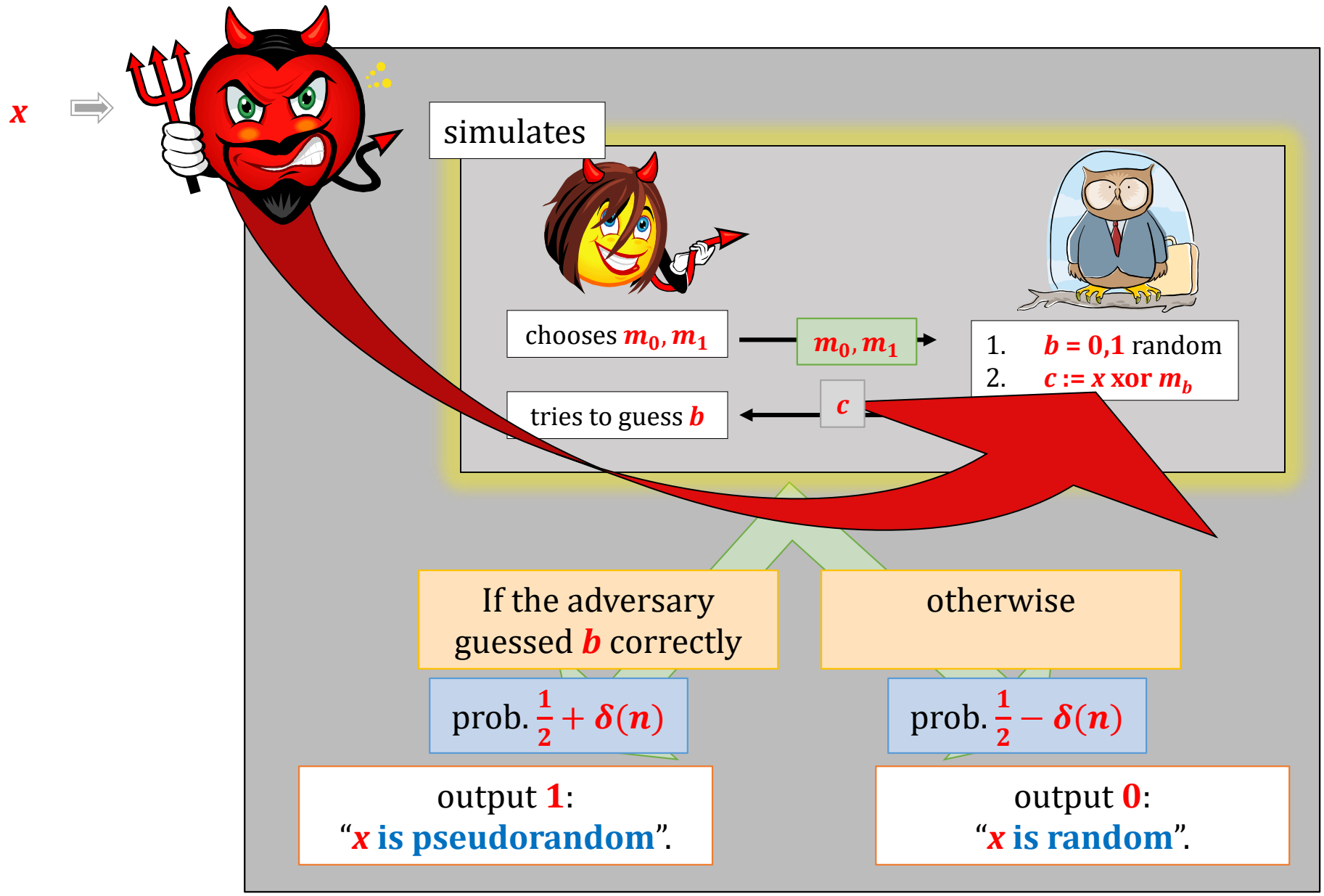
Suppose that it is **not** secure.
Therefore there exists an poly-time adversary that wins the "guessing game" with probability $\frac{1}{2} + \delta(n)$ where $\delta(n)$ is not negligible.


x
simulates
chooses m_0, m_1
m_0, m_1
1. $b = 0,1$ random
2. $c := x$ xor m_b
c
tries to guess b
If the adversary guessed b correctly
otherwise
output 1: "x is pseudorandom".
output 0: "x is random".

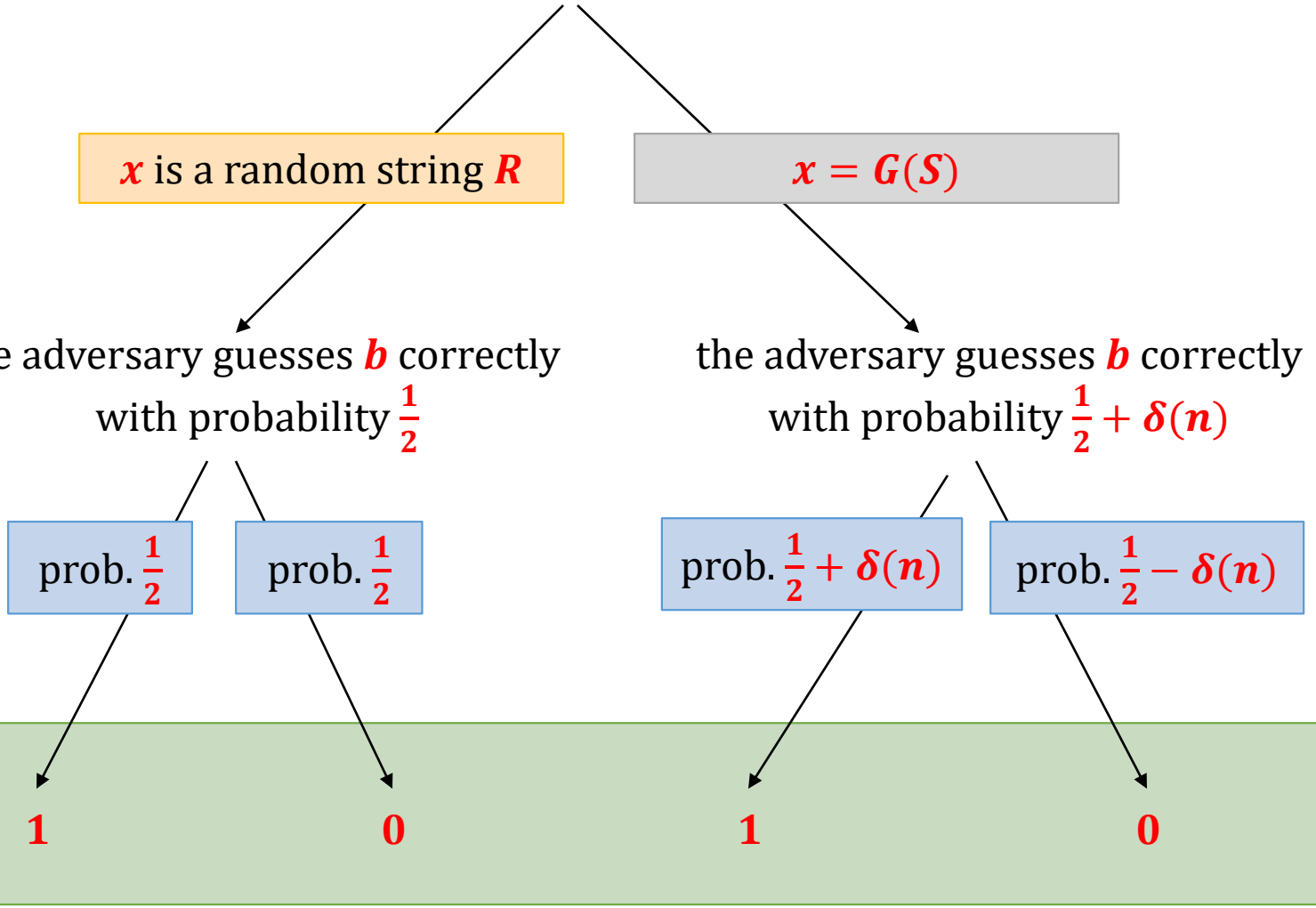
"scenario 0": x is a random string



"scenario 1": $x = G(S)$



Hence



$$|P(D(R) = 1) - P(D(G(S)) = 1| = \left|\frac{1}{2} - \left(\frac{1}{2} + \delta(n)\right)\right| = \delta(n)$$

Since δ is not negligible G cannot be a **cryptographic PRG**.

The complexity

The distinguisher  simply simulated

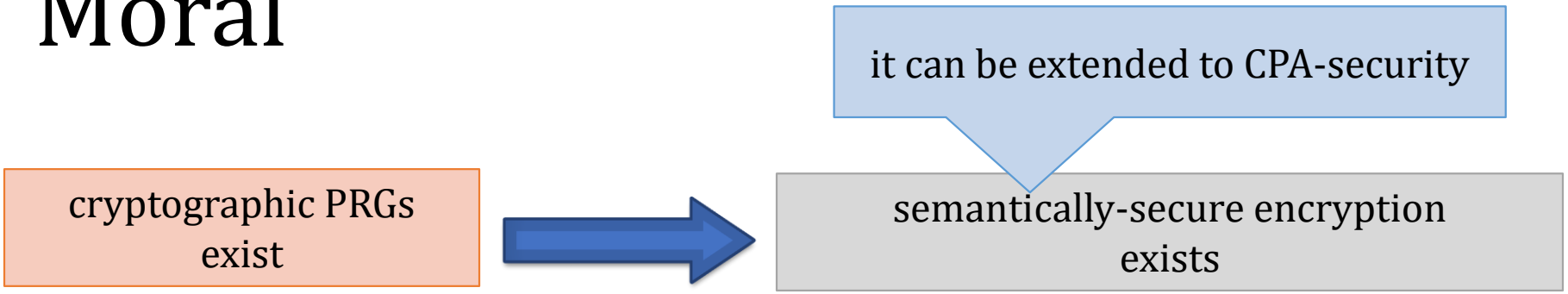
one execution of the adversary 

against the oracle  .

Hence he works in polynomial time.

QED

Moral



To construct secure encryption it suffices to construct a secure PRG.

Moreover, we can also state the following:

Informal remark. The reduction is tight.

A question

What if the distinguisher  needed to simulate

1000 executions of the adversary  ?

An (informal) answer

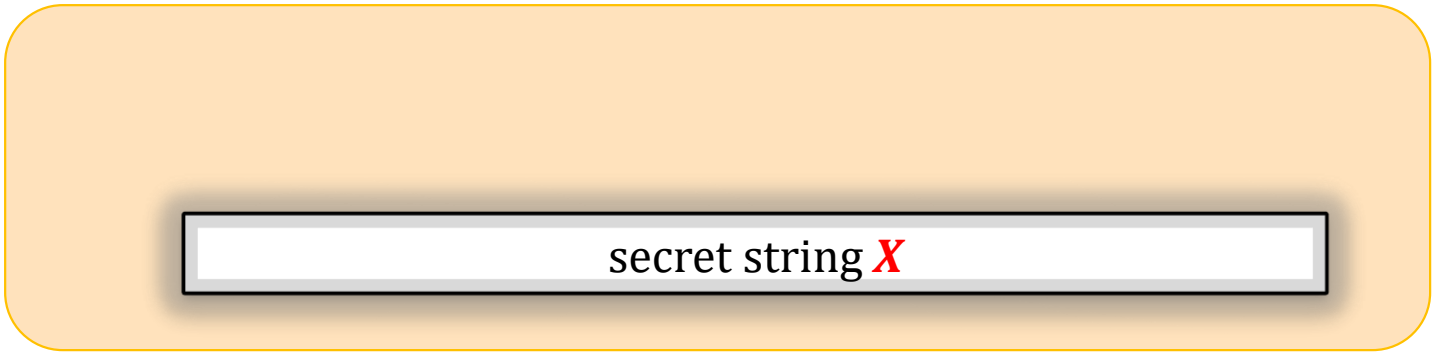
Then, the encryption scheme would be "**1000** times less secure" than the pseudorandom generator.

Why?

To achieve the same result  needs to work **1000** times more than  .

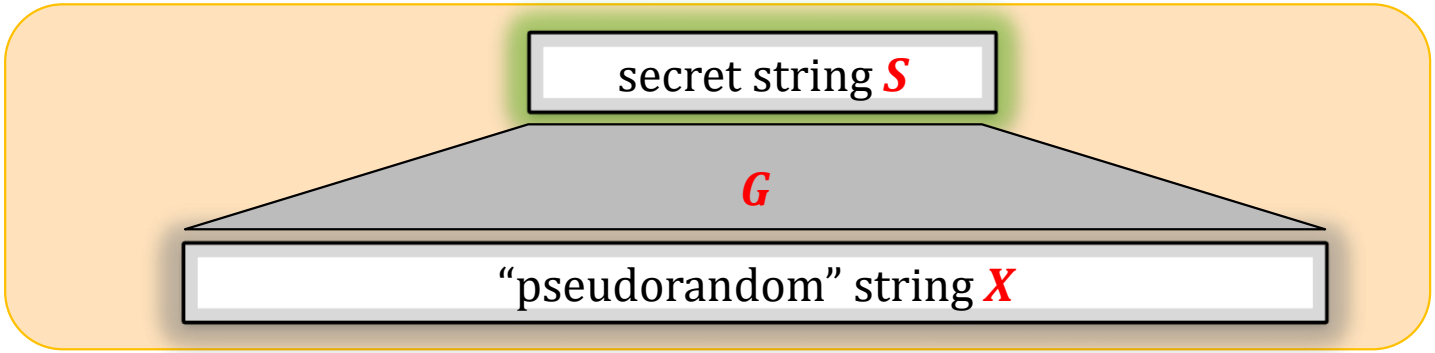
General rule

Take a secure system that uses some long secret string X.



Then, you can construct a system that uses a shorter string S, and expands it using a PRG:

$$X = G(S)$$



Constructions of PRGs

A theoretical result

a PRG can be constructed from any **one-way function**
(**very elegant**, **impractical, inefficient**)

Based on hardness of some
particular computational problems

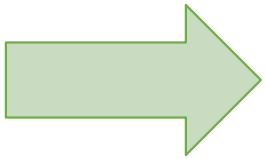
For example
[Blum, Blum, Shub. *A Simple Unpredictable Pseudo-Random Number Generator*]
(**elegant**, **more efficient**, still **rather impractical**)

"Stream ciphers"

ugly, **very efficient**, **widely used in practice**

Examples: RC4, Trivium, SOSEMANUK,...

Plan



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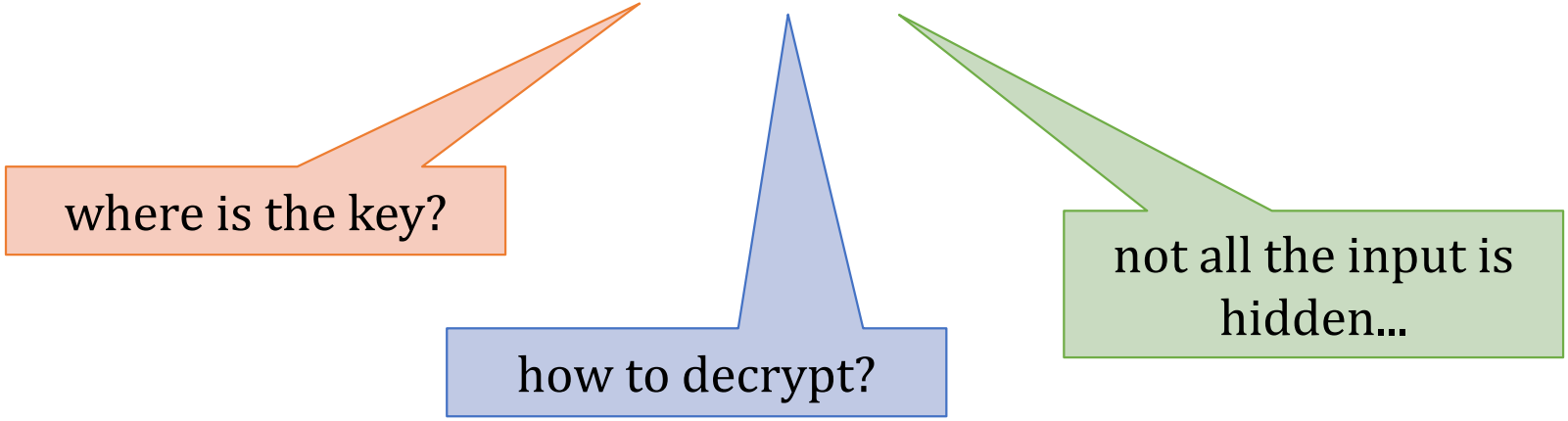
How to encrypt with one-way functions?

Naive (and wrong) idea:

1. Take a one-way function $\boldsymbol{f}$,
2. Let a ciphertext of a message $\boldsymbol{M}$ be equal to

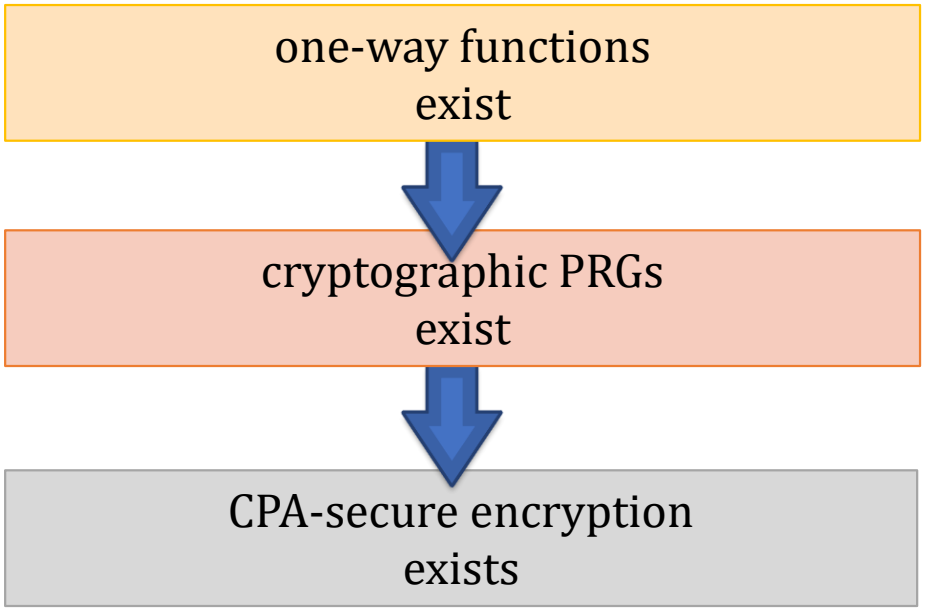
$$\boldsymbol{C := f(M)}$$



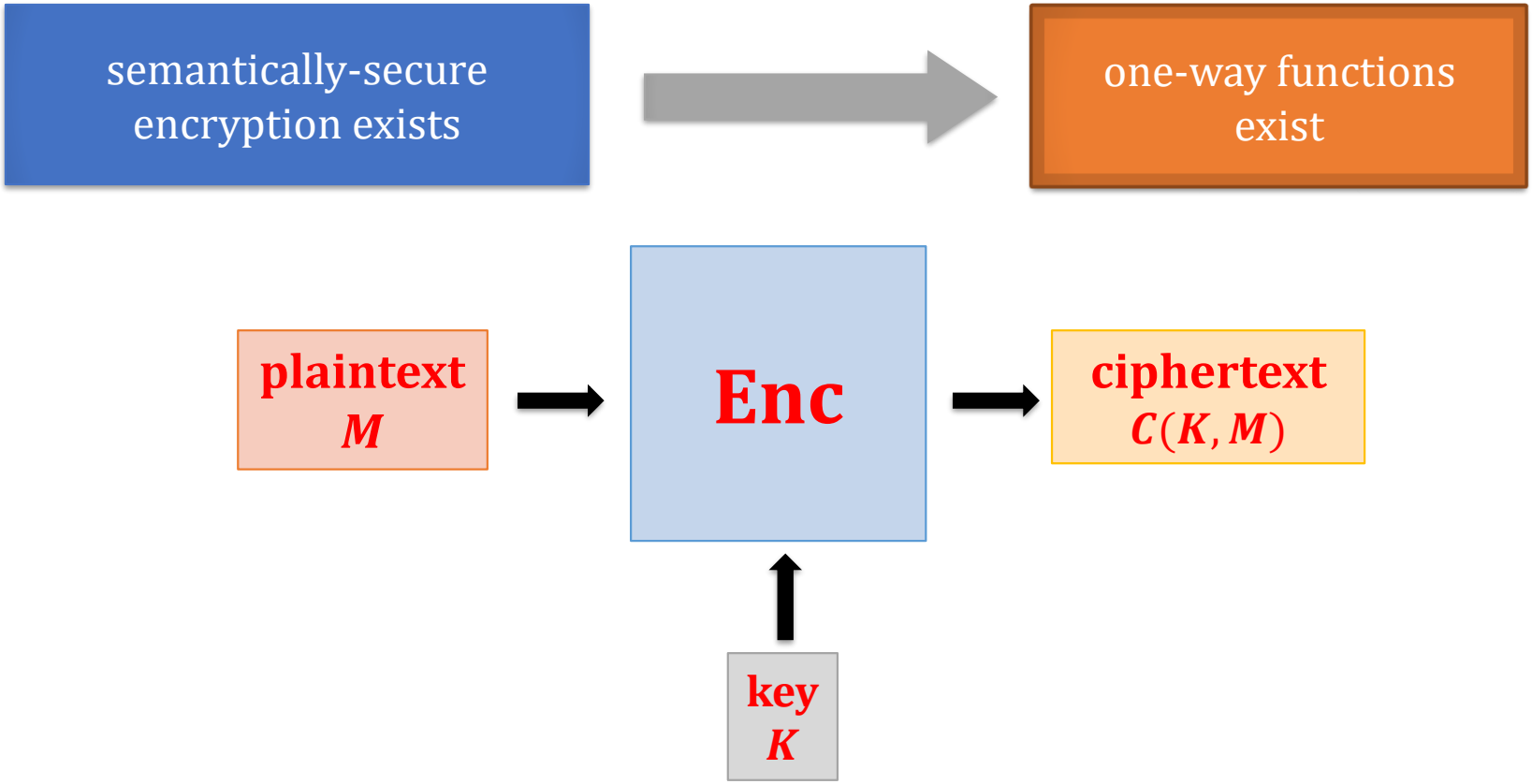
One of the most fundamental results in symmetric cryptography

[Håstad, Impagliazzo, Levin, Luby *A Pseudorandom Generator from any One-way Function*]:

"a PRG can be constructed from any **one-way function**"

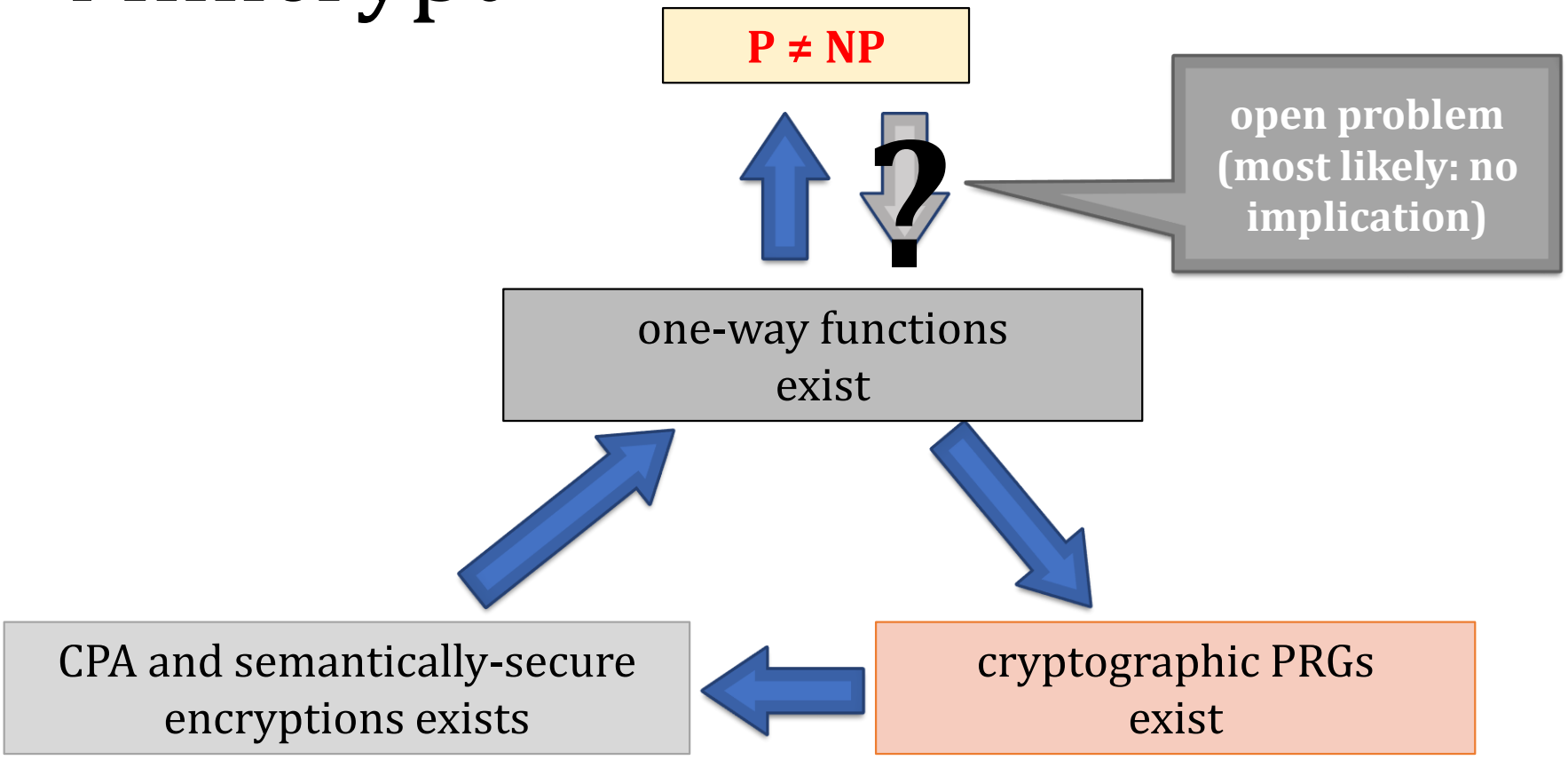


The implication also holds in the other direction



$f(K) = \mathbf{Enc}(K, (0, \ldots, 0))$ is a one-way function

"Minicrypt"



The "world" where the one-way functions exist is called "**minicrypt**".

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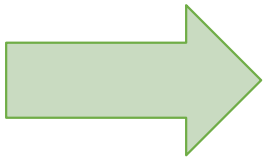
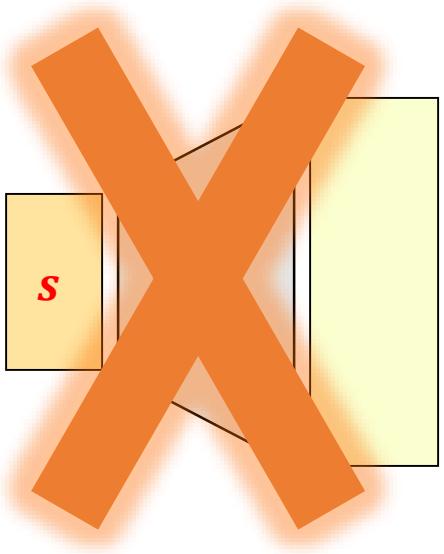
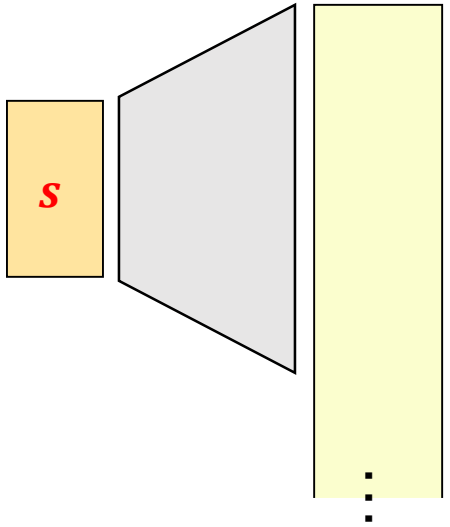


Stream ciphers

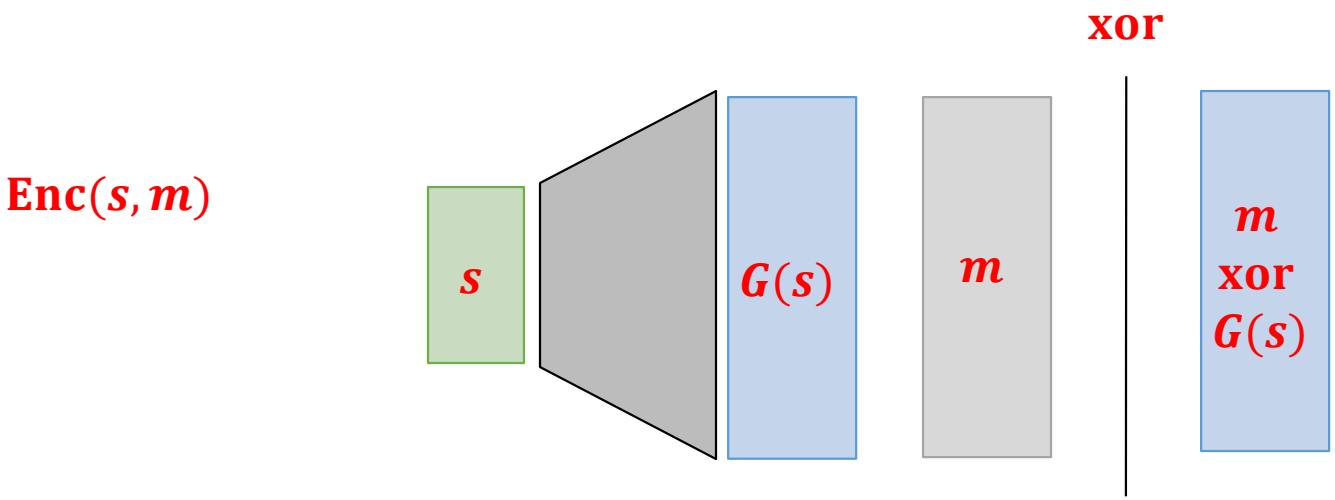
The pseudorandom generators used in practice are called **stream ciphers**





They are called like this because their output is an "infinite" **stream** of bits.

How to encrypt multiple messages using pseudorandom generators?



Of course we **cannot** just reuse the same seed
(remember the problem with the one-time pad?)

It is not just a theoretical problem!

Misuse of RC4 in Microsoft Office
[Hongjun Wu 2005]

RC4 – a popular PRG (or a "stream cipher")

> **"Microsoft Strong Cryptographic Provider"**
> (encryption in Word and Excel, Office 2003)

The key **s** is a function of a **password** and an **initialization vector**.

These values **do not change between the different versions** of the document!

Suppose **Alice** and **Bob** work together on some document:

$\mathbf{Enc}(s, m)$

$\mathbf{Enc}(s, m')$

The adversary can compute m xor m'

What to do?

There are two solutions:

1. The **synchronized mode**

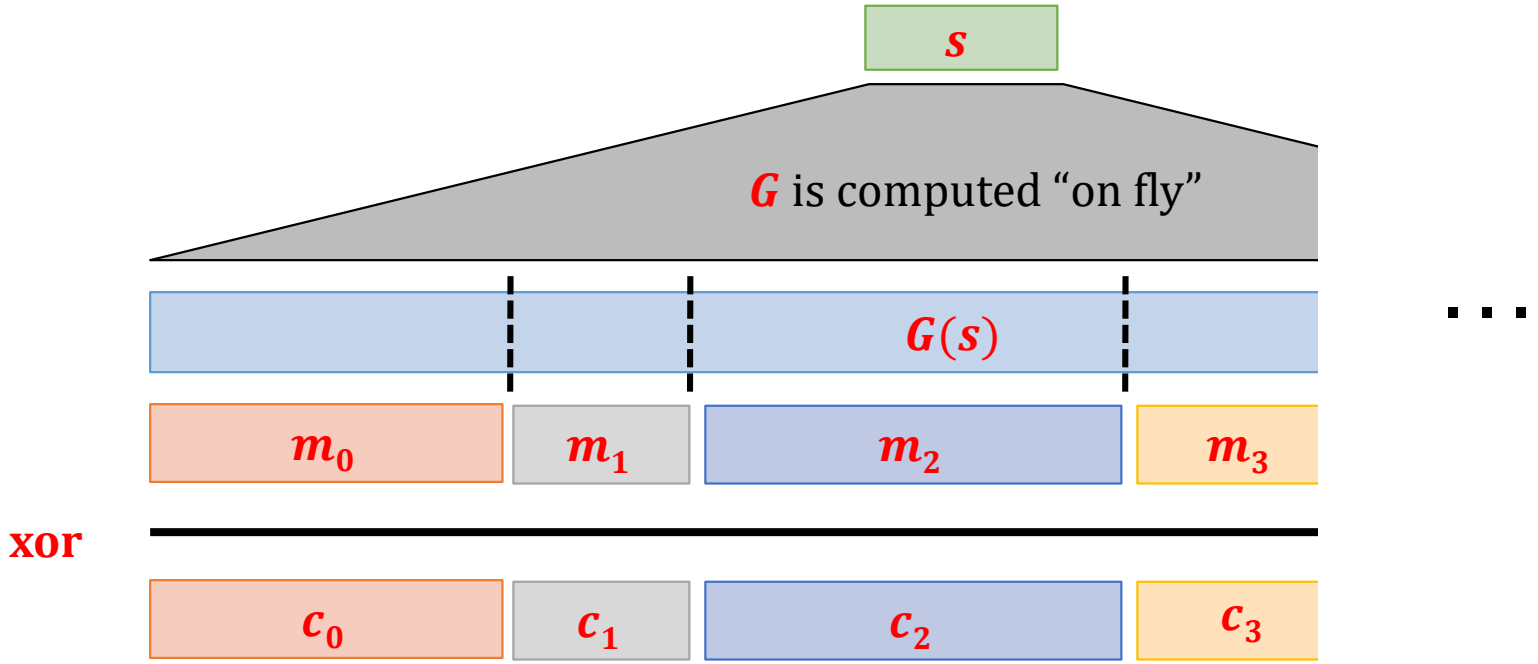
2. The **unsynchronized mode**

How to encrypt several messages

$G: \{0,1\}^n \rightarrow \{0,1\}^{\text{very large}}$ – a PRG.

this can be proven to be CPA-secure

divide $G(s)$ in blocks:



Unsynchronized mode

Idea

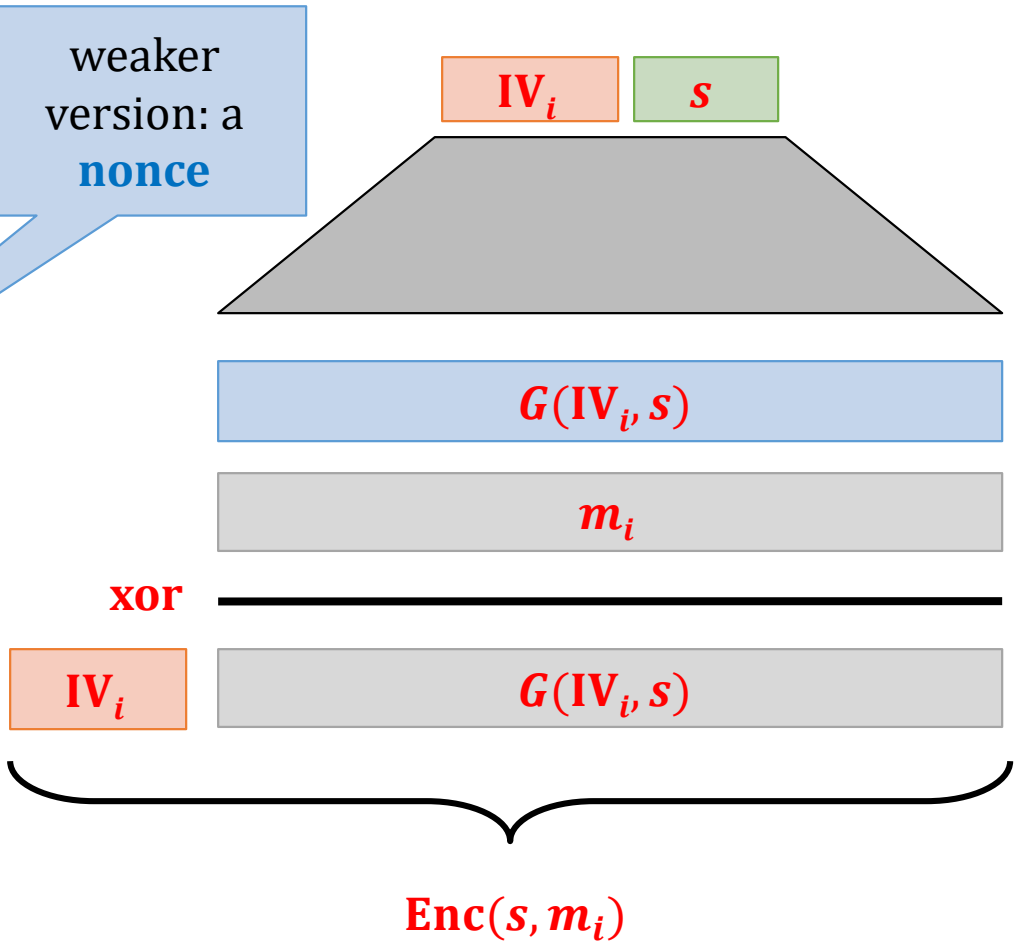
Randomize the encryption procedure.

Assume that G takes as an additional input

an **initialization vector** (**IV**).

The **Enc** algorithm selects a fresh random $\mathbf{IV}_i$ for each message $\mathbf{m}_i$.

Later $\mathbf{IV}_i$ is included in the **ciphertext**



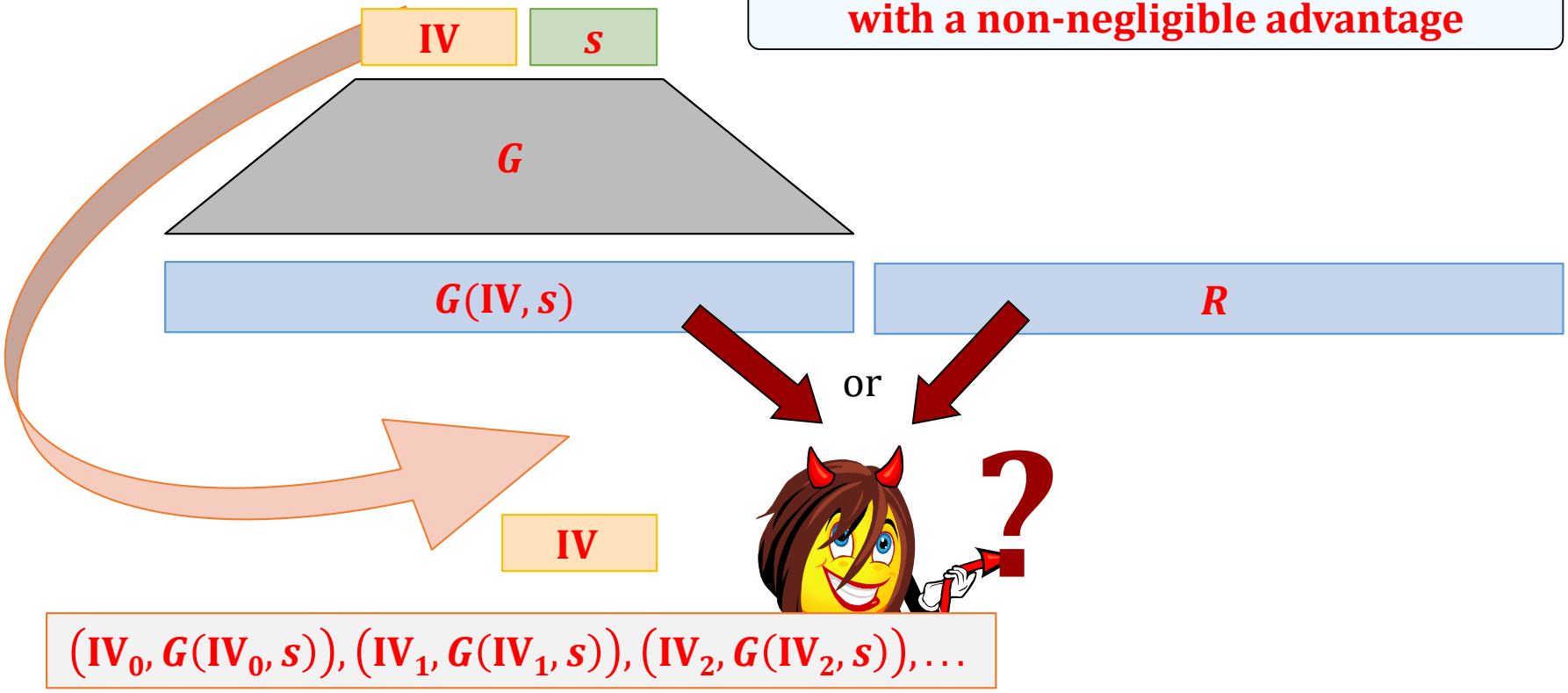
We need an "augmented" PRG

We need a **PRG** such that the adversary cannot distinguish $G(\mathbf{IV}, s)$ from a random string even if she knows **IV** and some pairs

$$\left(\mathbf{IV}_0, G(\mathbf{IV}_0, s)\right), \left(\mathbf{IV}_1, G(\mathbf{IV}_1, s)\right), \left(\mathbf{IV}_2, G(\mathbf{IV}_2, s)\right), \ldots$$

where $s, \mathbf{IV}, \mathbf{IV}_0, \mathbf{IV}_1, \mathbf{IV}_2 \ldots$ are random.

with a non-negligible advantage



How to construct such a PRG?

An **old-fashioned approach**:

1. take a standard **PRG** G
2. set $G'(\mathrm{IV}, s) := G(H(\mathrm{IV}, S))$

often:
just concatenate
IV and S

where H is a "hash-function" (we will define cryptographic hash functions later)

A more **modern approach**:

design such a G from scratch.

Popular historical stream ciphers

Based on the **linear feedback shift registers**:

- **A5/1** and **A5/2** (used in **GSM**)
  Ross Anderson:

  completely broken

  "there was a terrific row between the NATO signal intelligence agencies in the mid 1980s over whether GSM encryption should be strong or not. The Germans said it should be, as they shared a long border with the Warsaw Pact; but the other countries didn't feel this way, and the algorithm as now fielded is a French design."

- **Content Scramble System (CSS)** encryption

  completely broken

Other:

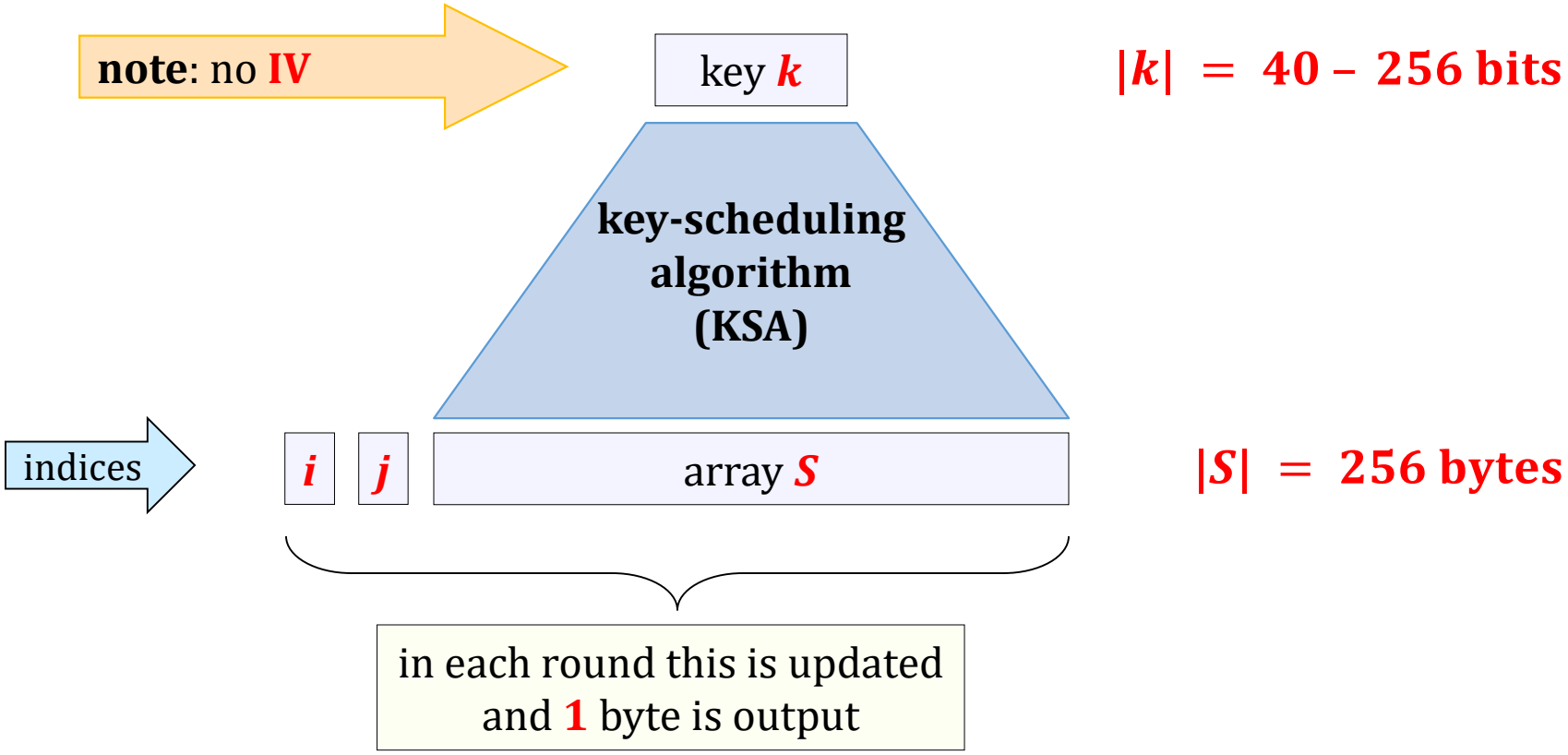
- **RC4**

  until recently very popular, but has serious security weaknesses

RC4

- Designed by **Ron Rivest** (**RSA Security**) in 1987.
  RC4 = "**Rivest Cipher 4**", or "**Ron's Code 4**".
- Trade secret, but in **September 1994** its description leaked to the internet.
- For legal reasons sometimes it is called: "**ARCFOUR**" or "**ARC4**".
- Used in **WEP** and **WPA** and **TLS**.
- **Very efficient and simple**, but has **security flaws**

RC4 – an overview



(this is called a **"pseudo-random generation algorithm (PRGA)"**)

RC4

KSA

```
for i from 0 to 255
        S[i] := i
end
for j := 0 for i from 0 to 255
        j := (j + S[i] + key[i mod 256]) mod 256
        swap(S[i],S[j])
endfor
```

PRGA

```
i := 0
j := 0
while GeneratingOutput:
        i := (i + 1) mod 256
        j := (j + S[i]) mod 256
        swap(S[i],S[j])
        output S[(S[i] + S[j]) mod 256]
endwhile
```

don't read it!

Problems with RC4

1. Doesn't have a separate **IV**.

2. It was discovered that some bytes of the output are **biased**.
   [Mantin, Shamir, 2001]

3. First few bytes of output sometimes **leak some information about the key**
   [Fluhrer, Mantin and Shamir, 2001]
   Recommendation: discard the first **768-3072** bytes.

4. Other weaknesses are also known...

Use of RC4 in WEP

- **WEP** = "Wired Equivalent Privacy"

- Introduced in **1999**, still widely used to protect **WiFi** communication.

- How **RC4** is used:

  to get the **seed**, the key k is **concatenated** with the **IV**

  - old versions: $|k| = 40$ **bits,** $|IV| = 24$ **bits** (artificially weak because of the **US export restrictions**)
  - new versions: $|k| = 104$ **bits,** $|IV| = 24$ **bits.**

RC4 in WEP – problems with the key length

- **|k| = 40 bits** is **not enough**:
  can be cracked using a **brute-force attack**
- **IV** is changed for each packet.
  Hence **|IV| = 24 bits** is also not enough:
  - assume that each packet has length **1500 bytes**,
  - with **5Mbps** bandwidth the set of all possible **IVs** will be exhausted in half a day
- Some implementations reset **IV := 0** after each restart – this makes things even worse.

see **Nikita Borisov, Ian Goldberg, David Wagner (2001). "*Intercepting Mobile Communications: The Insecurity of 802.11*"**

RC4 in WEP – the weak IVs

[Fluhrer, Mantin and Shamir, 2001]
(we mentioned this attack already)

For so-called **"weak IVs"** the key stream **reveals some information about the key.**

In response the vendors started to "filter" the **weak IVs**.

But then **new weak IVs were discovered**.

[see e.g. Bittau, Handley, Lackey *The final nail in WEP's coffin.*]

These attacks are practical!

[Fluhrer, Mantin and Shamir, 2001] attack:

Already in 2010: **Aircrack-ng** tool could break WEP in 1 minute (on a normal PC)

[see also: Tews, Weinmann, Pyshkin
Breaking 104 bit WEP in less than 60 seconds, 2007]

How bad is the situation?

RC4 is still rather secure if used in a correct way.

Example:

Wi-Fi Protected Access (**WPA**) – a successor of **WEP**:

several improvements (e.g. **128-bit key** and a **48-bit IV**).

RC4 forbidden in TLS

Internet Engineering Task Force (IETF)
Request for Comments: 7465
Updates: 5246, 4346, 2246
Category: Standards Track
ISSN: 2070-1721

A. Popov
Microsoft Corp.

February 2015

Prohibiting RC4 Cipher Suites

Abstract

This document requires that Transport Layer Security (TLS) clients and servers never negotiate the use of RC4 cipher suites when they establish connections. This applies to all TLS versions. This document updates RFCs 5246, 4346, and 2246.

Competitions for new stream ciphers

- **NESSIE** (New European Schemes for Signatures, Integrity and Encryption, 2000 – 2003) project **failed to select** a new stream cipher (all 6 candidates were broken)

  (where "*broken*" can mean e.g. that one can distinguish the output from random after seeing $\mathbf{2^{36}}$ bytes of output)

- **eStream** project (November 2004 – May 2008) chosen a portfolio of ciphers: **HC-128, Grain v1, Rabbit, MICKEY v2, Salsa20/12, Trivium, SOSEMANUK**.

Salsa 20

One of the winners of the **eStream** competition.

Author: Dan Bernstein.

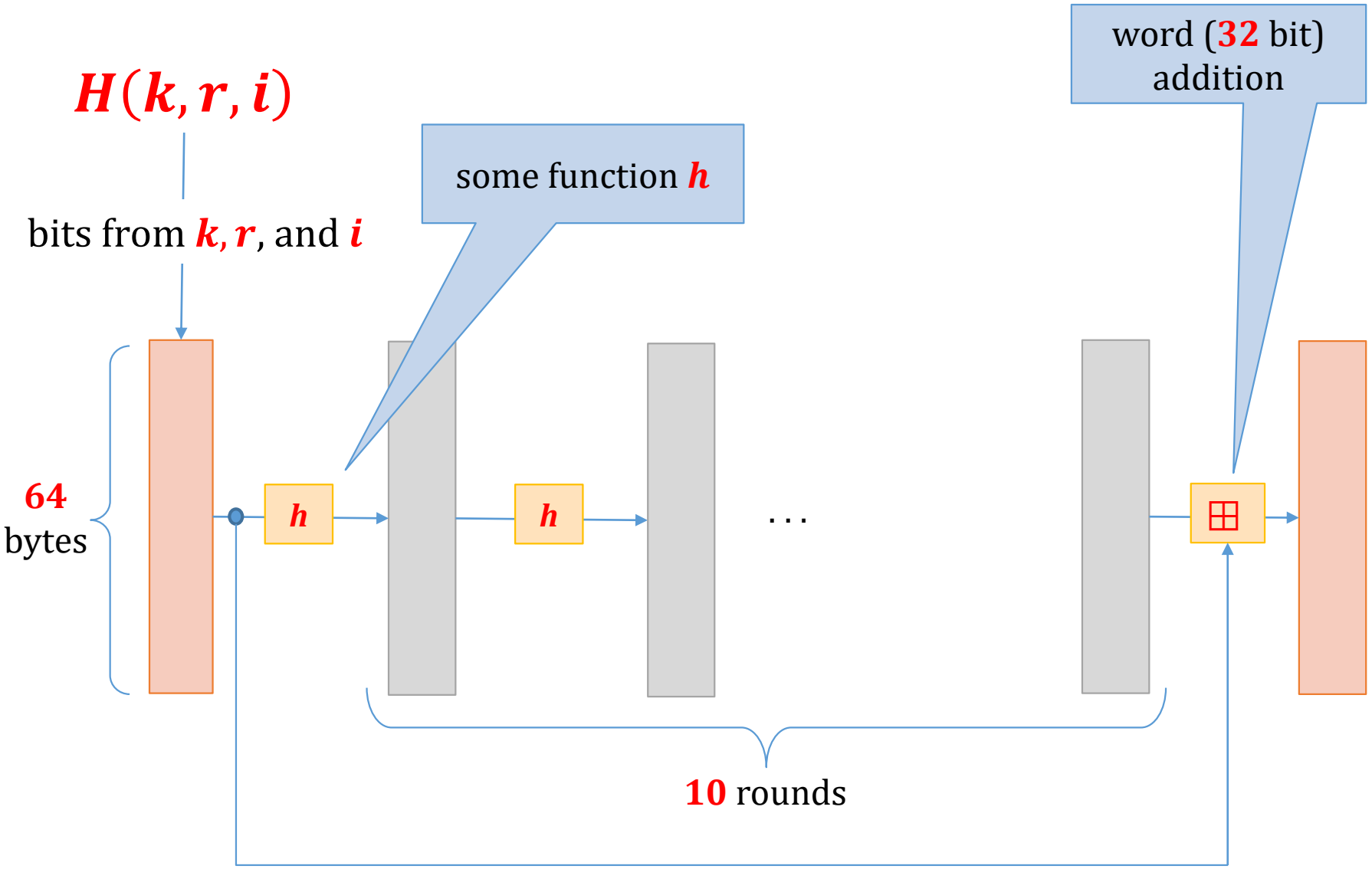
Very efficient both in **hardware** and in **software**.

key k
(size: **256** bits)

$$\mathbf{Salsa20}(k, r) := H(k, r, 0) || H(k, r, 1) || \cdots$$

nonce r
(size: **64** bits)

How is H defined?



Benchmarks

Algorithm	MiB/Second	Cycles Per Byte	Microseconds to Setup Key and IV	Cycles to Setup Key and IV
Salsa20/12	643	2.7	0.483	884
Sosemanuk	727	2.4	1.240	2269
RC4	126	13.9	2.690	4923

https://www.cryptopp.com/benchmarks.html

"All were coded in C++, compiled with Microsoft Visual C++ 2005 SP1 (whole program optimization, optimize for speed), and ran on an Intel Core 2 1.83 GHz processor under Windows Vista in 32-bit mode. x86/MMX/SSE2 assembly language routines were used for integer arithmetic, AES, VMAC, Sosemanuk, Panama, Salsa20, SHA-256, SHA-512, Tiger, and Whirlpool"

Is there an alternative to the stream ciphers?

Yes!

the **block ciphers**